Assignment I

BM-201 Advanced calculus.

Attempt any five.

Q1. Prove that a monotonically increasing sequence bounded above is convergent.

Q2. If $\sum_{n=1}^{\infty} a_n$ is a series of positive terms such that $\lim_{n\to\infty} n\left[\frac{a_n}{a_{n+1}} - 1\right] = l$, then prove that the given series is convergent if $l > 1$ and divergent if $l < 1$.

Q3. State and prove cauchy's second theorem on limits.

Q4. Test the convergence of following series:

$$\sum_{n=1}^{\infty} \left[(n+1)^{1/3} - n^{1/3}\right]$$

Q5. State and prove lagrange's mean value theorem

Q6 Show that $f(x) = x^2$ is uniformly continuous on $[-1, 1]$

Q7. Test the convergence of series

$$x \log x + x^2 \log 2x + x^3 \log 3x + \cdots \quad (x > 1)$$

Q8. Prove that $\sum_{n=1}^{\infty} (-1)^{n-1} \sin\frac{1}{n}$ is conditionally

BM-201. Advanced calulus.

Assignment II.

Attempt any five.

Q1. Examine for maximum and minimum values of the function : $x^2 - xy + y^2 + 3x - 2y + 1$.

Q2. Find the values of a and b so that

$$\lim_{x \to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3}$$ may be equal to 1.

Q3. Find minimum value of the function $u^2 + v^2 + w^2$ subject to the condition $au + bv + cw = a + b + c$.

Q4. Prove that $\int_0^1 \frac{dx}{\sqrt{1-x^n}} = \frac{1}{8}\sqrt{\frac{2}{\pi}}\left[\Gamma\left(\frac{1}{4}\right)\right]^2$.

Q5. Evaluate $\iiint (x+y+z)\,dx\,dy\,dz$ over the tetrahedran bounded by the planes $x = 0,\ y = 0,\ z = 0$ and $x + y + z = 1$.

Q6. Evaluate. $\iiint_{x^2+y^2+z^2 \le 1} x^2\,dx\,dy\,dz$.

Q7. Evaluate $\lim_{x \to \infty} \left(\frac{\pi}{2} - \tan^{-1}x\right)^{\frac{1}{x}}$.

Q8. Prove that $\beta(m, n) = \int^{\infty} x^{m-1} \ldots dx$.

BM-202. Differential equations

Assignment 1. | Do any five.

Q1. Show $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{3-x^2}{x^2}\sin x - \frac{3}{x}\cos x\right)$

Q2. Show that $\frac{d}{dx}F(\alpha,\beta,\gamma,x) = \frac{\alpha\beta}{\gamma}F(\alpha+1,\beta+1,\gamma+1,x)$

where $F(\alpha,\beta,\gamma,x)$ denotes hypergeometric function.

Q3. Evaluate $L(t^2\cos at)$.

Q4. Find the laplace transform of $\sinh 3t\cos^2 t$.

Q5. Solve $r+(a+b)s+abt = xy$ using monge's method. Here r, s, t have their usual meaning.

Q6. Solve the differential equation:

$$2x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + (1-x^2)y = 0$$ in series.

Q7. Solve the integral equation

$$f(t) = 1 + \int_0^t f(u)\sin(t-u)\,du.$$

Q8. Find inverse laplace transform of $\frac{1}{s(s+2)^3}$

BM-202: Differential equations

Assignment II

Do any five.

Q1. Find partial differential equation by eliminating the arbitrary function f from
$$f(x+y+z,\ x^2+y^2-z^2)=0$$

Q2. Solve $(D^3 + D^2D' - DD'^2 - D'^3)z = e^x \cos 2y$.

Q3. Find the shortest distance between the point $A(1,0)$ and the ellipse $4x^2+9y^2=36$.

Q4. Find extremals of the functional:
$$\int_{t_1}^{t_2} \sqrt{\dot{x}^2+\dot{y}^2}\,dt.$$

Q5. Find Singular integral of: $z = px + qy + \log pq$

Q6. Apply Charpit's method to find the complete integral of: $px + qy = pq$.

Q7. Show that Jacobi's condition is fulfilled for the functional $\int_{-1}^{1}(12xy + y'^2 + x^2)\,dx$ with fixed boundaries $A(-1,-2)$ and $B(1,0)$

Q8. Find extremal of $\int_{\theta_1}^{\theta_2} \sqrt{r^2+r'^2}\,d\theta$ where

Mechanics - (BM-203)

Assignment - I.

Attempt any five.

Q1. Define catenary. Also find intrinsic equation of the common catenary.

Q2. Find resultant of the systems of coplanar forces acting on a rigid body. Also write the condition of equilibrium

Q3. Find the equations of the central axis of any given system of forces acting on a rigid body

Q4. A heavy uniform rod rests with one end against a smooth vertical wall and with a point in ~~the~~ its length resting on smooth peg. Find the possition of equilibrium and show that it is unstable.

Q5 Prove that in case of catenary of uniform strength the mass per unit length is proportional to $\cosh\left(\frac{b}{c}\right)$

Q6. A heavy uniform cube of side $\frac{\pi r}{2}$ balances on the highest point of sphere of radius r. If there is no sliding, show that cube can rock through a right angle without falling

Q7. ~~If~~ Two forces P and Q are such that their central axis is given in position and the line of action of P is given. Show that the locus of the line of action of Q is a conicoid.

Q8. A ladder of length 2b and weight W has its centre of gravity at a distance $\frac{3}{4}b$ way up the ladder, stands on a smooth horizontal plane resting against a smooth vertical wall and middle point is tied to a point in the wall by a horizontal rope of length l. Find tension in the rope and reactions due to wall and horizontal plane.

Mechanics - (BM-203)

Assignment - II

Attempt any five

① Q- A particle of mass m is projected with constant velocity u in a resisting medium whose resistance varies as the velocity. To discuss its motion.

② Q→ If a planet were suddenly stopped in its orbit when at a distance 'a' from the sun, show that it would fall in the sun in time $\sqrt{2}\pi a^{3/2}/4\sqrt{\mu}$ which is $\sqrt{2}/8$ times the period of the planet's revolution.

③ Q→ If a particle is moving in a plane curve $r = f(\theta)$, then find the expressions for radial and transverse acceleration.

④ Q→ A particle moves with a central acceleration $\frac{\mu}{(\text{distance})^3}$. Find the path and distinguish the cases.

⑤ Q→ Derive the expressions of radial and transverse acceleration for the curve $r = f(\theta)$.

⑥ Q→ Derive differential equation of central orbit in polar form.

⑦ Q→ A particle moves along the curve $x = 4t$, $y = 6t - t^2$, find tangential and normal acceleration at $t = 3$.

⑧ Q→ Show that if the central acceleration varies as μu^n then there are at the most two apsidal distances.